PreCalculus
Review – Final Exam 2017

Name Mrs. Stirn

Calculators will be allowed on the entire exam. ☺

1. Find the remainder when $x^3 + 4x^2 + 7x - 9$ is divided by $x + 3$.

$-3 \mid 1 \;\; 4 \;\; 7 \;\; -9$
$\quad\;\; 0 \;\; -3 \;\; -3 \;\; -12$
$\quad\;\; 1 \;\; 1 \;\; 4 \;\; \boxed{-21}$

Remainder = −21

2. Find all the real and imaginary zeros.
$f(x) = (2x + 1)(x - 4)(x^2 + 7x - 3)$

$\frac{-7 \pm \sqrt{7^2 - 4(1)(-3)}}{2(1)}$

$-\frac{1}{2}, \; 4, \; \frac{-7 \pm \sqrt{61}}{2}$

3. Determine whether $(x - 2)$ is a factor of the polynomial $x^3 - 3x - 2$.

$2 \mid 1 \;\; 0 \;\; -3 \;\; -2$
$\quad\;\; 0 \;\; 2 \;\; 4 \;\; 2$
$\quad\;\; 1 \;\; 2 \;\; 1 \;\; \boxed{0}$

yes → no remainder

4. Find all the real and imaginary zeros.
$h(x) = x^3 - 10x^2 + 44x - 69$

$3 \mid 1 \;\; -10 \;\; 44 \;\; -69$
$\quad\;\; 0 \;\; 3 \;\; -21 \;\; 69$
$\quad\;\; 1 \;\; -7 \;\; 23 \;\; \boxed{0}$

$\frac{7 \pm \sqrt{(-7)^2 - 4(1)(23}}{2(1)}$

$3, \; \frac{7 \pm i\sqrt{43}}{2}$

5. Determine whether $k = 2$ is an upper bound for the real zeros of the function $f(x) = -2x^3 - 4x^2 + x - 2$. Answer YES or NO. Show work.

$2 \mid -2 \;\; -4 \;\; 1 \;\; -2$
$\quad\;\; 0 \;\; -4 \;\; -16 \;\; -30$
$\quad\;\; -2 \;\; -8 \;\; -15 \;\; -32$

No – there are negatives in bottom row

6. Give the y-intercept, zero(s) and equation(s) of vertical asymptote(s).

$g(x) = \frac{x + 4}{x^2 - x - 6}$ $\quad \frac{(x+4)}{(x-3)(x+2)}$

$(x=0)$

y-int: $\frac{4}{-6} = -\frac{2}{3}$

Zero: $(-4, 0)$

VA: $x = 3$, $x = -2$

7. A cubic polynomial $f(x)$ has zeros $4 + 2i$ and -5. Write its equation if $f(3) = 20$.

$4+2i$ } sum $= 8$
$4-2i$ } prod $= 16 - 4i^2 = 20$

$y = a(x+5)(x^2 - 8x + 20)$

$20 = a(8)(5) \quad a = \frac{1}{2}$

$y = \frac{1}{2}(x+5)(x^2 - 8x + 20)$

8. Give the removable discontinuity or state that there are none.

$h(x) = \frac{x^2 + 4x - 5}{x^2 - 25}$ $\quad \frac{(x+5)(x-1)}{(x+5)(x-5)}$

$(-5, \; 3/5)$

9. Write the polynomial in factored form.
$f(x) = x^4 + 3x^3 - 3x^2 + 3x - 4$

$1 \mid 1 \;\; 3 \;\; -3 \;\; 3 \;\; -4$
$\quad\;\; 0 \;\; 1 \;\; 4 \;\; 1 \;\; 4$
$-4 \mid 1 \;\; 4 \;\; 1 \;\; 4 \;\; \boxed{0}$
$\quad\;\; 0 \;\; -4 \;\; 0 \;\; -4$
$\quad\;\; 1 \;\; 0 \;\; 1 \;\; \boxed{0}$

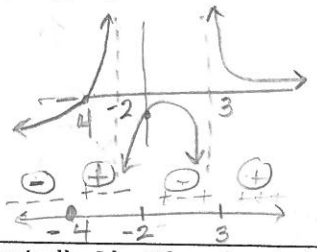
$f(x) = (x^2 + 1)(x - 1)(x + 4)$

10. Sketch a graph and identify the limits.

$g(x) = \frac{x + 4}{x^2 - x - 6}$ $\quad \frac{x+4}{(x-3)(x+2)}$

$\lim_{x \to 3^+} g(x) = \infty$

$\lim_{x \to -2^-} g(x) = \infty$



11. Solve the equation algebraically. Identify any extraneous solutions. $x + 2 = \frac{15}{x}$

$x(x+2) = \left(\frac{15}{x}\right)x$

$x^2 + 2x = 15$

$x^2 + 2x - 15 = 0$

$(x+5)(x-3) = 0$

$x = -5, \; 3$

12. Solve the equation algebraically. Identify and exclude any extraneous solutions.

$x(x+1)\left(2 - \frac{1}{x+1}\right) = \left(\frac{1}{x^2 + x}\right)x(x+1)$

$x(x+1)$

$2x(x+1) - x = 1$

$2x^2 + 2x - x = 1$

$2x^2 + x - 1 = 0$

$(2x - 1)(x + 1) = 0$

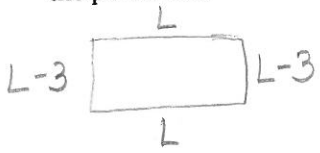
$x = \frac{1}{2}, \; x = -1$ Extr.

$x = \frac{1}{2}$

13. The length of a rectangle is 3 cm more than the width. Write an equation that gives the length as a function of the perimeter.

$L = w + 3$

$L - 3 = w$



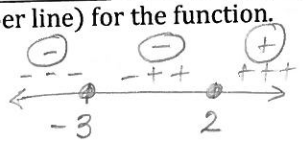
$P = 4L - 6$

$P + 6 = 4L$

$\frac{P+6}{4} = L$

14. Make a sign chart (number line) for the function.

$f(x) = (x - 2)(x + 3)^2$



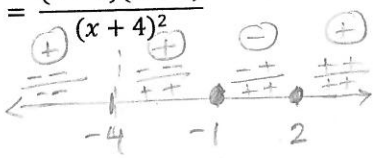
Use the chart to give the interval(s) where $f(x) \geq 0$.

$\{-3\}$ and $[2, \infty)$

single point $f(x) = 0$

15. Make a sign chart (number line) for the function.

$$g(x) = \frac{(x - 2)(x + 1)}{(x + 4)^2}$$



Use the chart to give the interval(s) where $g(x) < 0$.

$(-1, 2)$

16. State whether the function represents exponential growth or decay. Then give the initial value and growth or decay factor.

A. $h(x) = 0.954\,(1.067)^x$ — growth

initial value: 0.954

growth factor: 6.7%

B. $g(x) = 1.35(0.851)^x$ — decay

initial value: 1.35

decay factor: 14.9%

17. Write an exponential growth function that has initial value 47.5 and is increasing at a rate of 16.2% per year.

$f(x) = 47.5\,(1.162)^x$

18. Write an exponential decay function that has initial value 16.2 and is decreasing at a rate of 1.17% per month.

$g(x) = 16.2(0.9883)^x$

19. The 1990 population of Lakeville was 17,450 and it is increasing at a rate of 2.17% each year.

$f(x) = 17{,}450\,(1.0217)^x$

A. Estimate the population in 2012.

$x = 22$

$f(x) = 17{,}450(1.0217)^{22} = 27{,}984$

B. In what year would the population be about 32,000?

$32{,}000 = 17{,}450(1.0217)^x$

$x = 28.24$ (from calc)

In 2018

20. A substance used in chemistry has a half-life of 5 days. If there are 60 grams present now, how much will there be after 4 full weeks?

$f(x) = 60\left(\frac{1}{2}\right)^{t/5}$

4 weeks = 28 days

$f(x) = 60\left(\frac{1}{2}\right)^{28/5}$

$f(x) = 1.24$ grams

21. A house that has been increasing in value at a rate of 2.25% per year is currently worth $195,000. How much was it worth 10 years ago?

$195{,}000 = A_0(1.0225)^{10}$

$195{,}000 = 1.2492\,A_0$

$156{,}099.48

10 years ago

22. Condense the log expression.

$2\log_3 6 - 2\log_3 2 + \log_3 3$

$\log_3 6^2 - \log_3 2^2 + \log_3 3$

$\log_3 \frac{36}{4} + \log_3 3$

$\log_3 (9 \cdot 3) = \log_3 27 = 3$

23. Expand the log expression.

$\log \sqrt{\frac{x^3y^4}{w}} = \log \left(\frac{x^3y^4}{w}\right)^{1/2}$

$\frac{1}{2}\log\left(\frac{x^3y^4}{w}\right)$

$\frac{1}{2}(\log x^3 + \log y^4 - \log w)$

$\frac{1}{2}(3\log x + 4\log y - \log w)$

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24. Solve the equations.

A. $\log_3(2x+4) = 4$

$3^4 = 2x+4$

$2x+4 = 81$

$2x = 77$

$x = 38.5$

B. $\log_4(5) + \log_4(2x+1) = \log_4(3x+10)$

$\log_4[5(2x+1)] = \log_4[3x+10]$

$10x+5 = 3x+10$

$7x = 5$

$x = 5/7$

25. Solve the equations.

A. $16^{x+2} = 32$

$(2^4)^{x+2} = 2^5$

$4x+8 = 5$

$4x = -3$

$x = -3/4$

B. $7^{3x} = 28$

$\log 7^{3x} = \log 28$

$3x \log 7 = \log 28$

$3x = 1.7124$

$x = 0.571$

26. Suppose you invest $3,500 at 3½% interest compounded monthly. How much money will you have after 5 years?

$A = 3500\left(1 + \frac{.035}{12}\right)^{12 \cdot 5}$

$A = \$4,168.40$

27. An investment of $6,500 grew to $8,000 when 3.5% interest was compounded quarterly. How long did it take?

$8000 = 6500\left(1 + \frac{.035}{4}\right)^{4t}$

$1.2308 = \left(1 + \frac{.035}{4}\right)^{4t}$

$\log 1.2308 = 4t \log\left(1 + \frac{.035}{4}\right)$

$4t = 23.83$

$t \approx 6$ yrs

28. Express in degrees: $\frac{7\pi}{6}$

$\left(\frac{7\pi}{6}\right)\left(\frac{180}{\pi}\right) = 210°$

29. Express in radians: −150°

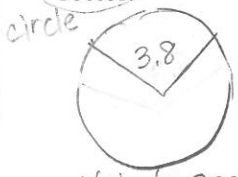
$\frac{-150°}{1} \cdot \frac{\pi}{180} = -\frac{5\pi}{6}$

30. Give a positive and negative angle, each coterminal with the angle given. (Coterminal angles end in exactly the same place.)

A. 110° (use degrees) $(\pm 360°)$ 470°, −250°

B. $-\frac{5\pi}{12}$ (use radians) $(\pm 2\pi)$ $-\frac{29\pi}{12}, \frac{19\pi}{12}$

31. The central angle of a sector of a circle is 3.8 radians, and the arc length is 11.4 units. Find the radius of the sector.



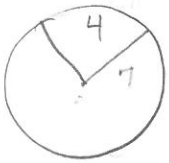
Not to scale

$S = r\theta$

$11.4 = r(3.8)$

$r = 3$ units

32. The arc length of a sector of a circle is 4 inches and the radius is 7~~cm~~ in. Find the central angle measure in degrees.



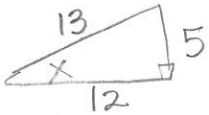
Should be same units.

$S = \frac{\theta}{360} \cdot 2\pi r$

$4 = \frac{\theta}{360} \cdot 2\pi(7)$

$\theta = 32.74°$

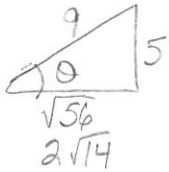
33. Find the measure of the smallest angle of a 5-12-13 right triangle.



$\tan x = \frac{5}{12}$

$x = 22.6°$

34. If θ is an acute angle of a right triangle and $\sin\theta = \frac{5}{9}$, find the values of the other five trigonometric functions.



$\sin\theta = 5/9$ $\quad \csc\theta = 9/5$

$\cos\theta = \sqrt{56}/9$ $\quad \sec\theta = 9/\sqrt{56}$

$\tan\theta = 5/\sqrt{56}$ $\quad \cot\theta = \sqrt{56}/5$

35. Find the value of sec (48°).

$\cos 48° = 0.669$

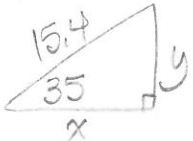
$\sec 48° = 1.494$

36. Find the value of cot (2.65).

$\tan 2.65 = -0.535$

$\cot 2.65 = -1.868$

37. A right triangle has one acute angle that measures 35°, and the hypotenuse is 15.4. Find the lengths of the two legs of the triangle.

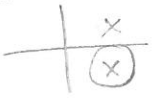


$\cos 35° = \frac{x}{15.4}$ $\quad x = 12.6$

$\sin 35° = \frac{y}{15.4}$ $\quad y = 8.8$

38. Given an angle A in standard position, give the quadrant that is described.

A. $\sec A > 0, \cot A < 0$

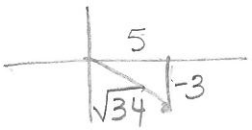


Quad 4

B. $\csc A < 0, \tan A > 0$

Quad 3

39. An angle passes through (5, -3). Give the value of all six trigonometric functions.



$\sin\theta = -3/\sqrt{34}$
$\cos\theta = 5/\sqrt{34}$
$\tan\theta = -3/5$
$\csc\theta = -\sqrt{34}/3$
$\sec\theta = \sqrt{34}/5$
$\cot\theta = -5/3$

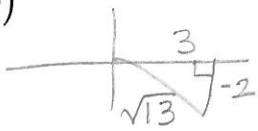
40. Give the amplitude, period, horizontal shift, vertical shift, maximum value, minimum value.

$y = 4 \sin \pi (x - \frac{1}{2}) + 3$

Amp: 4
Per: 2
Horiz: ½ right
vert: up 3
max: 7
min: -1

41. Evaluate: $\sec\left(\tan^{-1}\left(-\frac{2}{3}\right)\right)$

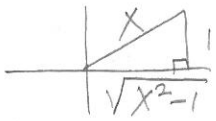
$\frac{\sqrt{13}}{3}$



42. Write an algebraic expression for the following.

$\cot\left(\sin^{-1}\left(\frac{1}{x}\right)\right)$

$\sqrt{x^2-1}$



43. Simplify the following:

A. $\frac{\sin A}{1-\cos^2 A}$ $\frac{\sin A}{\sin^2 A} = \frac{1}{\sin A} = \csc A$

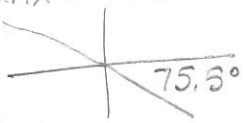
B. $\sec A \cdot \tan A (1 - \sin^2 A)$ $\frac{1}{\cos A} \cdot \frac{\sin A}{\cos A} \cdot \cos^2 A = \sin A$

C. $\cot^2 A \cdot \sec A \cdot \tan A$ $\frac{\cos^2 A}{\sin^2 A} \cdot \frac{1}{\cos A} \cdot \frac{\sin A}{\cos A} = \frac{1}{\sin A} = \csc A$

44. Find all solutions for $0 \le x < 360°$.

$\cot x = -0.258$

$\tan x = -3.876$

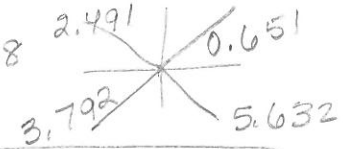


104.5°, 284.5°

45. Find all solutions for $0 \le x < 2\pi$.

$\sin^2 x = 0.367$

$\sin x = \pm 0.6058$



0.651, 2.491, 3.792, 5.632

You will also need to know how to answer questions related to the Ferris wheel graph, and answer questions related to a sine / cosine graph. Last few quiz reviews and Olympiad reviews should help.